Adult Learner Program • 89-11 Merrick Blvd., Jamaica, NY 11432
E-mail: alpref@queenslibrary.org • Web: www.queenslibrary.org/tasc-transition

Module 3: Workshop 7
Lesson Plan

Overall Learning Goals Strategies for Developing Common Core Skills in Content Areas (Math & Science): to train administrators and adult educators to develop instructional strategies for developing Common Core skills in content areas to better serve their ESOL, ABE, and pre-HSE student constituency.	**Lesson Topic** Reviewing the 8 standards of mathematical practice and depth of knowledge & cognitive complexity, including selecting, creating, and using rich, rigorous mathematical tasks and concept-based lessons.
Curriculum Developer Tyler Holzer **Workshop Trainer**	**Date** **Location**

Intended Audience

- **Instructors** (content was designed as a workshop for Instructors).
- Note: Sample student material is included for Instructors to analyze during the workshop. Instructors may also use sample student materials in their classes.

Standards Alignment

The Common Core Standards for Mathematical Practice:

- MP1: Make sense of problems and persevere in solving them
- MP2: Reason abstractly and quantitatively
- MP3: Construct viable arguments and critique the reasoning of others
- MP4: Model with Mathematics
- MP5: Use appropriate tools strategically
- MP6: Attend to precision
- MP7: Look for and make use of structure
- MP8: Look for and express regularity in repeated reasoning

Goals and Objectives (SWBAT)

- Participants will become familiar with the Common Core Standards for Mathematical Practice and how they apply to ABE/HSE instruction in the Common Core era.
- Participants will understand how to make appropriate shifts in their classrooms so that they can pose cognitively demanding tasks that build both perseverance and deep understanding of key mathematics topics.
- As a result, instructors will be able to use their textbooks as jumping-off points for creating mathematically rich lessons that involve focused problem-solving activities, and for developing open-ended problems that ask students to think beyond the "answer" to a problem. In the end, participants will have the tools to think deeply about the mathematical tasks they select for classroom use, and they will ensure that they are engaging all learners in their class by utilizing content-rich lessons.
- Specifically:
    - Participants will know the 8 Standards for Mathematical Practice.
    - Participants will know Webb's Depth of Knowledge (DOK) and apply DOK rubric to sample TASC questions.
    - Participants will know the components of a mathematically rich problem that will develop the Standards for Mathematical Practice.
    - Participants will know how to turn traditional textbook problems into open-ended, rigorous tasks
    - Participants will know how to develop content-rich lesson plans for use in the ABE/HSE classroom

Warm-Up/Review

- The facilitator will welcome participants and encourage them to sit at tables with other participants. The workshop will rely heavily on group discussion and work.

The TASC Transition curriculum is a collaborative project of the New York State Education Department and the Queens Borough Public Library, supported by funding from the New York State Department of Labor.

- The facilitator will pose the question to the group: “What are the characteristics of a good math student?”
  - These questions will draw out specific behaviors that appear in the Standards for Mathematical Practice, particularly sense-making, perseverance, precision, and working strategically.
    - Some sample answers from a previous workshop are: Good math students can identify the problem, persevere in solving, think outside the box, organize information, analyze similarities and differences, talk about math, analyze evidence, be efficient, enjoy being challenged, feel okay about being wrong, do their homework, show up to class on time, ask questions, participate in group work, and help their classmates.
  - Each table should spend five to ten minutes working together to answer these questions, and they should record their results.
    - The facilitator will lead a whole-group discussion about the characteristics of good math students and will ask each table what they came up with. As groups offer their characteristics, the facilitator will record them on chart paper and post them in the room.
    - To close the warmup, the facilitator will lead a brief discussion of good student behaviors and make connections between these behaviors and the cognitive demands presented by good math problems.
- As a follow-up, if time allows, the facilitator should posed the question: “What are the characteristics of a good math problem?”
  - As participants volunteer characteristics of good math problems, the facilitator will help forge connections between good problems and good student behaviors.
    - Some sample responses include: A good problem engages learners at all levels; illustrates an important mathematical concept; requires both computational fluency and conceptual understanding; has connections to other topics; requires students to think deeply; can be approached in more than one way; promotes classroom discussion; and has a “low entry” and “high ceiling” (i.e., is not difficult to understand but allows for a number of extension questions and deeper inquiry).
  - The facilitator will explain that in this workshop, we will examine the mathematical behaviors that, through effective instruction, we can hope to inspire in our students. As the workshop goes on, we will draw explicit connections between good mathematical behaviors and rigorous, mathematically rich problems.

References (APA Style)

- Doyle, W. (1988). Work in mathematics classes: The context of students' thinking during instruction. *Educational Psychologist,23*, 167–80.
- Hiebert, J., & Stigler, J. W. (2004). “A world of difference: Classrooms abroad provide lessons in teaching math and science. (2004). Journal *of Staff Development*, *25*, 10–15.
- *Illustrative Mathematics.* (2015). Retrieved from https://www.illustrativemathematics.org
- Kabiri, M. S., & Smith, N. L. (2003). Turning traditional textbook problems into open-ended problems. *Mathematics Teaching in the Middle School*, *9*, 186–92.
- Lambert, R., & Stylianou, D. A. (2013). Posing cognitively demanding tasks to all students. *Mathematics Teaching in the Middle School*, *18*, 500–6.
- Petit, M., & Hess, K. K. (2006). Applying Webb’s depth of knowledge and NAEP levels of complexity in mathematics. The National Center for Assessment.
- Schwan Smith, M., & Stein, M. K. (1998). Selecting and creating mathematical tasks: From research to practice. *Mathematics Teaching in the Middle School*, *3*, 344–50.
- Standards for Mathematical Practice (2015). Retrieved from http://www.corestandards.org/Math/Practice/.

Technology and Handouts

Technology Needs	**Presentation Needs & Handouts**
• AV cart with projector, laptop, and speakers will be provided. • Laptop or tablet computer for each student with access to Internet. • Latest version of Adobe Flash installed on laptops.	• Chart Paper. • Markers. Each item listed below will be available in PDF format. • Creating an Open Ended Problem. • DOK Rubric Handout. • DOK Problem Pairs. • Lesson Plan Template. • Mathematical Practice Bulleted. • Reflecting on the Standards for Mathematical Practice. • Reflecting on Webb. • The Aquarium Problem.

	• The Aquarium Problem for Students. • The Aquarium Problem Lesson Plan. • Turning Traditional Textbook Problems into Open-Ended Problems. • What's My DOK? • Selecting and Creating Mathematical Tasks. • Posing Cognitively Demanding Tasks to All Students.

Lesson Plan Activities

Part 1: Unpacking the Standards for Mathematical Practice

Lesson Content

Participants will explore the Standards for Mathematical Practice and their connection to problem-solving situation, the TASC, and good student behaviors.

Lesson Materials

Handout on the Standards for Mathematical Practice, adapted so that the components of each standard are clearly spelled out in a bulleted list.

Questions to Answer

- What are the Standards for Mathematical Practice?
- What implications do they have for my teaching?
- What implications do they have for students preparing for the TASC?
- How are the Standards for Mathematical Practice reflected in the work students are able to do in a problem-solving situation?

Opening/Background

- Following the warm-up discussion of good student behaviors and good math problems, the facilitator will distribute the Standards for Mathematical Practice handout and lead a brief introduction and discussion of each standard.
    - From the Common Core State Standards Initiative: "The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important 'processes and proficiencies' with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (i.e., comprehension of mathematical concepts, operations, and relations), procedural fluency (i.e., skill in carrying out procedures flexibly, accurately, efficiently, and appropriately), and productive disposition (i.e, habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy)."
    - The facilitator should carefully walk through each of the Standards for Mathematical Practice, offering clarifications on what each of the standards mean.
    - Some general examples for each standard follow.
        - **MP1**: More advanced students might transform an algebraic expression to get information they need; less advanced students might rely on using physical objects or drawings to help them understand and solve a problem.
        - **MP2**: Learners can understand the relationships between problem situations and mathematical notation; students are able to move comfortably between the two representations; students can create a coherent representation of the problem at hand.
        - **MP3**: Beginning students can construct arguments using concrete references, such as drawings; advanced students can determine domains to which an argument applies.
        - **MP4**: Students might use geometry solve a design problem, or use a function to analyze how one quantity of interest depends on another
        - **MP5**: Students are able to use technological tools (online resources, graphing calculators, etc.) or classroom resources (protractors, rulers, etc.) to solve problems.
        - **MP6**: Students can talk clearly and efficiently with peers about mathematical processes and solution methods; students are also precise in their mathematical computations. They specify units of measurement and label axes on graphs.
        - **MP7**: Beginning students might notice that 8 groups of 5 is the same amount as 5 groups of 8. Students can factor a quadratic equation by noticing that the two factors of the constant term should have a sum equal to the middle term.

The TASC Transition curriculum is a collaborative project of the New York State Education Department and the Queens Borough Public Library, supported by funding from the New York State Department of Labor.

- **MP8**: Students can draw advanced conclusions based on repeated work.

Teacher Focus

Questions for discussion: "Which of these standards do you find that your students have the most challenge achieving?" "Which of these standards is the most challenging to address as an ABE/HSE teacher?

- The unique challenge for teachers of adults is that many of us only see our students twice a week for roughly six hours. Developing good mathematical practice in students can be difficult when we don't have as much classroom time.
- Moreover, our students often have a number of demands on their time outside of academic ones, and they may not have time for homework. Teachers should reflect on how to maximize the productive use of in-class time to build good mathematical behaviors in their students.

Lesson Activities

- Activity 1: The facilitator will divide the participants into groups or pairs, depending on the number of participants.
  - Each group will be assigned either one or two of the Standards for Mathematical practice—if there are a small number of participants, some groups may be assigned two or more standards.
  - In this case, the facilitator should allow extra time for the activity so that each standard can be considered and discussed.
- Activity 2: Groups/Pairs will be instructed to think of a specific problem-solving situation in which a student would need to employ the Standards of Mathematical Practice that you/your group has been assigned. They should be given at least fifteen to twenty minutes, and groups should be encouraged to think of multiple classroom/problem-solving situations for each standard.
  - The facilitator should direct participants to be very clear about what the activity would entail. What kinds of problems and group work would it involve, and how would the activity promote the Standards for Mathematical Practice?
  - After the groups have had ample time to think of situations to which each of their group's standards would apply, they should present their findings to the larger group.
  - Participants should present on chart paper. The participants' examples of problem-solving situations should be posted around the room.
  - Many of the problem-solving situations that arise will likely touch on more than one of the Standards for Mathematical Practice. As each group wraps up its brief presentation, the facilitator should encourage all participants to see connections across the standards and discuss them.

Wrap Up/Assessment

- The facilitator will direct participants back to the group's notes from the warm-up discussion about the characteristics of a good math student.
  - For closing discussion: "How does the list of good student behaviors align with the Standards for Mathematical Practice? Do some of these behaviors apply to more than one standard?"
  - This should be a brief discussion, with the instructor soliciting responses and allowing participants to do most of the talking and reflecting.
- For further assessment, the facilitator will give out the reflection handout for the Standards of Mathematical Practice.
  - Participants should be given ten minutes to reflect individually and write a response. The facilitator may choose to collect participant responses for review during the break or between workshops, but all participants' responses should be returned to them.

Part 2: Exploring Webb's Depth of Knowledge

Lesson Content

Participants will analyze Webb's Depth of Knowledge (DOK), its implications in the Common Core math classroom, and connections to the Standards for Mathematical Practice.

Lesson Materials

- Handout of Webb's DOK as it applies to mathematics.
- Handout of sample assessment questions for DOK analysis.

Questions to Answer

- What is Webb's DOK?
- How do sample assessment questions meet specific DOK criteria?
- Why is it crucial that we know and understand Webb's Depth of Knowledge, and how does it relate to our work in preparing students to take and pass the TASC mathematics subtest?

- How can Webb's DOK help us design and utilize meaningful classroom tasks?

Opening/Background

- Facilitator will distribute handout on Webb's DOK and provide background information. Webb's DOK is a framework for analyzing mathematical tasks, classroom activities, and problems in terms of their cognitive demand and rigor. Webb's DOK can help teachers to think carefully about their classroom tasks and make sure that students are engaging in meaningful work.
  - Webb's DOK was instrumental in the creation of the TASC, and the objective structure of the TASC is based on the DOK level of the questions in each subtest. The goal of the TASC is to be testing exclusively at DOK levels 2 and 3 by 2016. Up to that point, there has been a gradual increase in the DOK levels of most questions on the test.
  - **DOK 1 (Recall):** Students are performing routine operations or recalling simple definitions. At this level, the teacher is often simply conveying information to students through telling or demonstrating. Students are memorizing, restating, and describing terms and procedures. A DOK 1 question, for example, would ask students to choose the correct definition of a geometric shape.
  - **DOK 2 (Skills and Concepts):** At DOK 2, the teacher asks questions of the student and facilitates their interaction with a problem. The student makes decisions, solves problems, uses models, and applies mathematics.
  - **DOK 3 (Strategic Thinking):** At this level, the teacher continues to be a facilitator between the student and the concepts underlying the mathematics. The teacher is a resource for the student; she asks questions, observes, and guides. The student thinks deeply, asks questions, assesses information, and tests hypotheses.
  - **DOK 4 (Extended Thinking):** DOK 4 requires extended thinking over what could be a long period of time. Here, the teachers supports and extends student thinking. Students are making mathematical discoveries on their own and evaluating results.
  - Not all problems will fit squarely into a DOK level, and participants should be aware that in the discussions and analyses to follow, there may be some disagreement. This is to be expected.
- The facilitator will discuss how the DOK applies to the TASC.
  - The 2014 TASC and its accompanying Readiness Assessment were primarily DOK level 1, with some questions at level 2.
  - The 2015 TASC and Readiness Assessment 2 have more questions at DOK 2, with fewer level 1 questions, but some level 3 questions. The new constructed-response questions are good examples of what might be asked of a student at DOK 3.
  - The 2016 TASC will be primarily DOK levels 2 and 3, with little or no level 1 questions. Because it requires thinking and exploration over a period of time, DOK level 4 is not tested by the TASC or other HSE exams.
- Participants should be given time to read through the entire DOK handout on their own before the activity begins, and the facilitator should check with participants to make sure that everything is clear before moving on.

Lesson Activities

- Activity 1: The facilitator will distribute the handout What's My DOK? to participants at each table/group. Using the DOK rubric, participants will determine the problem's DOK level and provide evidence from the rubric to support their choice. Participants should work on all six problems individually for five to ten minutes, and then they should work in small groups to discuss their findings.
  - If some participants/groups disagree about a questions DOK level, the whole group should discuss this and attempt to come to a consensus.
    - **Problem 1**: This is a level 1 problem. It asks the student to recall a simple definition.
    - **Problem 2**: This is a level 3 problem. To answer the question successfully, students must know the definition of a function, but they must also analyze the argument of another student and draw a conclusion about its validity. Moreover, the question goes a step further by asking students to generalize a rule.
    - **Problem 3**: This is a level 1 problem, though some participants may say that it is level 2. The problem is very simple and is not connected to a real-world application. In order for this to be a level 2 or 3 problem, students would need to analyze the meaning of the answer, or put it in context.
    - **Problem 4**: This is a DOK 2 problem. Extending a sequence falls under DOK 2 on the rubric.
    - **Problem 5**: This is a DOK 2 problem, but is a case to be made that it is level 3. On the rubric, translating between language and algebra falls under both levels. Because this question is relatively straightforward and does not ask the student to extend or explain their thinking, it probably fits best under DOK 2.
    - **Problem 6**: This is a DOK 3 or 4 problem. This question requires students to do mathematical investigation, formulate a hypothesis, and test it. Here, the students are revealing a powerful mathematical idea—that the sum of the interior angles of a polygon can be found by using the formula $180(n - 2)$—on their own. This problem is adapted from Hiebert and Stigler's "A World of Difference" and is included here because it will

gesture toward Workshop 13.

- Activity 2: During the activity, the facilitator will guide groups toward specific language on the DOK rubric.
  - o The facilitator will check in with each group/pair, asking them why they chose the DOK level that they did.
  - o For extended thinking, the facilitator may ask participants to consider how, for example, a DOK level 1 questions could be transformed into a DOK level 2 or 3 questions
- Activity 3: After groups have had time to analyze the problems, the facilitator will lead the whole group in a discussion of the DOK levels, making sure to spend time analyzing problems that might border on two DOK levels.
  - o If this happens, the facilitator should point to language from the handout that clarifies which DOK level is correct.
  - o In the discussion, participants should point to specific aspects of the problem that cause it to fall into a particular DOK level.

Wrap Up/Assessment

- The facilitator will lead brief closing discussion of the benefits of focusing instruction around problems at DOK levels 2 and 3. Possible questions for discussion: "Why would it be worthwhile to spend more time on fewer tasks? Which makes more sense: introducing a topic with a DOK 1 task and working up to DOK 3/4, or vice-versa?
  - o Answers to these questions will vary, and there is not necessarily one correct answer. The big takeaway is that students should engage with tasks at all DOK levels because they will encounter DOK 1, 2, and 3 questions on the TASC. In designing classroom tasks, teachers should ensure that they move above DOK 1 and 2 whenever possible, so that students are engaging on a deeper level with important mathematical concepts.
- For assessment of connections between lessons 1 and 2, the facilitator should ask participants to reflect individually on how the Standards for Mathematical Practice discussed earlier connect to Webb's DOK. They should write their responses on notebook paper so that they can refer back to them later. The facilitator may also choose to give participants the handout Reflecting on Webb's DOK. This can be collected or not, but it should be returned to participants.
  - o Prompt for assessment: "What connections do you see between the Standards for Mathematical Practice and Webb's Depth of Knowledge?

Lesson Part 3: Using Open-Ended Problems and Mathematically Rich Tasks

Lesson Content

Participants will continue discussing mathematically rich tasks, and they will learn about the characteristics of an open-ended math problem. Participants will continue DOK analysis by looking at an example of a rote textbook problem and creating an adaptation of that problem that invites further investigation and is more mathematically rigorous.

Questions to Answer

- How can a standard textbook problem be adapted into a more rigorous task?
- How does it benefit teachers to spend time focusing on more involved, DOK 2 or 3 problems and developing problem-solving skills?
- How does this connect to the Standards for Mathematical Practice?

Lesson Materials

- Handout of the Aquarium Problem: Two Versions.
- Handout on Creating an Open-Ended Problem.

Opening/Background

- The facilitator should provide a brief background on what constitutes an open-ended problem. An open-ended problem is one in which the solution pathway might not be immediately obvious, and so the problem is more cognitively demanding than a traditional textbook problem. A problem is called open-ended when it has more than one acceptable solution, or when there are multiple entry points and solution methods. Open-ended problems often require students to discuss their choices or their justification for believing that their solution is the correct one.
  - o According to Kabiri and Schwan (2003), open-ended problem-solving activities can help teachers to "meet the needs of all learners in their class…When teachers present these types of problems, they not only support the high achievers but also communicate high expectations and provide opportunities for higher-level thinking for all students in the class."
  - o This suggests that using open-ended problems can make classroom management easier, and it allows all students to contribute to whole-class discussions. This strategy "provides enrichment or sponge activities for students who finish early and are ready for more challenge while giving slower learners the time they need as well."
- The facilitator will begin by distributing handout of the Aquarium Problem: Two Versions. If possible, the facilitator may choose to distribute the entire Kabiri and Smith article, which contains the Aquarium Problem and accompanying

analysis.

- The participants will solve the traditional version of the problem. This will take only a short time. Here, the facilitator should ask participants why the problem could be solved so quickly. Participants should be asked to reflect on how demanding the problem was to solve, and the facilitator could record responses to the following questions on chart paper.
  - What skills did you have to use to solve this problem?
  - What new skills did you learn from solving this problem?
  - What was the DOK level of this problem?
- Next, participants will solve the open-ended version of the problem. They should be allowed to work for at least five to ten minutes, and the facilitator should check in with everyone while they are working, asking each participant to further explain their rationale for the size of their aquarium. Participants should be given ample time to think the problem through.
  - Participants who finish while others are still working should be directed to discuss their solution with others.
- The facilitator will lead a discussion about the open-ended version of the problem, asking the same questions as the last time. This time, the facilitator should ask the participants what they liked about this problem. If the facilitator recorded responses to the first problem on chart paper, he/she should repeat the process here.
  - This version of the problem touches on specific Common Core content standards, and it promotes more of the Standards for Mathematical Practice than the traditional version. Standards addressed are:
  - **CCSS.MATH.CONTENT.6.G.A.2**: Solve real-world and mathematical problems involving area, surface area, and volume
  - **CCSS.MATH.CONTENT.HSG.MG.A.1** and **CCSS.MATH.CONTENT.HSG.MG.A.3**: Apply geometric concepts in modeling situations
  - The problem also requires students to **make sense of problems and persevere in solving them (MP1)**, **construct a viable argument about the best dimensions for the aquarium (MP3)**, and **model with mathematics (MP4)**.
- Participants will be invited to share their thoughts on how classroom time could be better spent on the open-ended version of the problem. It should be noted that students will need to solve problems like the traditional version, but by focusing on the open-ended version of the problem, students get the knowledge they need to solve simple problems while learning how to talk about mathematics in context.

Lesson Activities

- Activity 1: The facilitator will distribute the handout Creating an Open-Ended Problem.
  - Participants will be instructed to first solve the problem, and then to write an open-ended version of the problem that would fall under DOK 2 or 3.
  - Participants will work independently for ten to fifteen minutes. While participants are thinking about possibilities for more rigorous problems, the facilitator should float around the room and check in with each participant. If participants come up with an open-ended problem quickly, they should be directed to create other problems. The facilitator should also direct participants to extend the question into other content areas.
  - For the facilitator: The handout contains a simple chart that asks participants to find the total enrollment for a particular year. If participants are having time coming up with ideas for an open-ended version of the problem, or for possible extensions, they could be prompted with the following questions. This list is by no means exhaustive; it should be used to help participants generate ideas if they are stuck or are having difficulty finding multiple entry points into the problem.
    - Could you ask a question about a future year? Could you make predictions about the enrollment in 2016? What else could you ask to draw out student thinking?
    - Do you notice a trend in the growth? What are some other ways that students might represent this?
    - What could be some reasons why the program is growing? What might happen that would cause it to have fewer classes?
    - What if there are students taking both ABE and ESOL?
    - What might the student demographic be? Is there anything there that you could explore?
    - How many students are usually in a class? Could you look into a question about staffing or budgeting?
    - What if you made up data about another school? Could a student use the data to choose which program to join?
    - What if we were tasked with separating the total enrollment numbers into separate classes? What might be taught in those classes? Would there be any overlap?
    - Could this information be represented in a graph? What would that graph look like?
- Activity 2: After participants have had ample time to work individually, they should share their problems with a partner

or in a small group.
    - Groups should work to solve all the open-ended versions of the problem.
    - Once the group members have had time to collaborate, they should present one or two versions of the new problems on chart paper.
- Activity 3: The facilitator will ask groups to talk about their open-ended problems will invite discussion on what the group likes about the problem.
    - The facilitator will guide participants back to the DOK rubric and ask each group to find a descriptor showing why their new problem is DOK 2 or 3.

Teacher Focus

Questions for discussion: "What was challenging about this activity? As teachers, why might it be useful to think about building up existing textbook questions and turning them into open-ended questions?"

Wrap Up/Assessment

- The facilitator will lead a brief discussion about the activity.
    - Students need to be prepared to answer a variety of different questions within a content area. Teaching rigorous, open-ended problems and problems that require further investigation allows students to struggle productively, talk about their thinking, and develop the background knowledge needed to prepare the kinds of questions that will appear on the TASC.
    - DOK level 2 and 3 questions often invite multiple solution methods, which gives instructors the opportunity to expose students to solution pathways that are different from their own, and in turn develops mathematical fluency and problem-solving ability.
- The facilitator should distribute the Problem Pairs handout for further assessment. Participants should solve both problems and reflect on the experience of solving each.
    - In the classroom, participants should use one or more of the Problem Pairs with their students. Participants should have a discussion with the class in which their students reflect on which of the two problems was more challenging and why.
    - In class, teachers should talk about why the second version of each problem is more closely aligned with the kind of material students will see on the TASC.

Lesson Part 4: Planning a Lesson around an Open-Ended Problem

Lesson Content

Participants will examine ways to turn a rigorous problem-solving activity into the centerpiece of a lesson or topic.

Questions to Answer

- What is the best balance between extended problem-solving activities and workbook-type problems?
- Why should we make sure to expose our students to both types of problems?
- How can a lesson effectively combine problem-solving activities with workbook problems?
- What implications does this have for lesson planning?

Lesson Materials

Blank lesson plan template

Opening/Background

- The facilitator will discuss how an extended problem-solving activity can be used as the introduction to or centerpiece of a lesson. The facilitator will make connections to Webb's Depth of Knowledge and to the Standards of Mathematical Practice.
- The facilitator will ask participants to refer back to the Aquarium Problem from the previous activity.
    - The facilitator will lead the group in a brainstorming session in which participants list the mathematical skills that students would develop by working on this problem. The group will consider the background knowledge students might need to complete this task and collectively attempt to situate the problem within a larger lesson.
    - The facilitator will also lead the group in brainstorming what a lesson involving the Aquarium Problem might look like. Question for discussion: "Let's assume your class has just finished working on the Aquarium Problem, and students have been given time to discuss their solutions. If this were your classroom, where would the lesson go next?"
        - The aquarium problem promotes repeated use of the formula for finding the volume of a rectangular solid ($V = lwh$), but perhaps more important, it asks students to think critically about an object taking up a three-dimensional space. This offers some interesting possibilities for the analysis of other three-dimensional shapes, like cylinders and spheres.

- The problem works well as an introduction to volume, and most lessons based upon this problem would further explore volume. Participants may choose to use this problem as a jumping-off point in which the class looks at other three dimensional shapes, or it might also lead them to solving a higher-level volume/surface area problem.
- Participants should feel free to suggest multiple pathways from the Aquarium Problem into other new content and problems. Each participant's classroom likely follows a different structure, and responses to the discussion question will vary and should promote lively discussion.

Lesson Activities

- Activity 1: The facilitator will distribute a blank lesson plan template to each participant.
  - The facilitator will ask participants to think about a 3 hour class in which volume is introduced via the Aquarium Problem.
  - Participants should be given ten to fifteen minutes to work individually in thinking about their lesson.
  - Participants should consider the following possibilities in creating the lesson.
    - What kind of activator could be used in advance of the problem? That is, how could the teacher get students in the mindset of thinking about three dimensional shapes? What would that activator look like?
    - How much time would students need to work on the Aquarium Problem? How much time should be set aside for discussion and sharing after students have finished working?
    - What would small group work look like? Should students share their findings about the Aquarium Problem before the whole-class discussion?
    - What new material would the teacher introduce once the class is finished discussing the problem?
    - Would there be another problem-solving activity within the same lesson?
    - What might assessment for this activity look like?
    - What other materials would students need?
    - For extended thinking, what kind of problem-solving activity might follow this one as part of another lesson?
- Activity 2: After working individually to brainstorm ideas for a lesson, participants should work at their tables to find consensus and write the outline of a lesson on chart paper.
- Activity 3: When participants have finished outlining the lesson, the facilitator will ask groups to volunteer to present their lesson to the other participants.
  - While groups are presenting, the facilitator should ask clarifying questions about why they chose to take the lesson in the direction that they did.

Wrap Up/Assessment

- To wrap up the activity and assess the participants, the facilitator will ask participants what might be the best balance of problem-solving activities and textbook problems.
  - Question for reflection: "Now that you have created the outline of a lesson, how would you supplement it with more traditional TASC-type problems? In terms of time, what would be the best balance between the two?"
  - Participants will take five to ten minutes to reflect on and write about how open-ended problems would fit into their classroom, and they will share their responses with the group.
- Using either the lesson they created, or one of the lessons created by another group, participants should try the lesson in the classroom.
  - The facilitator will also distribute the sample lesson plan based on the Aquarium Problem for participants to consider as they develop an activity that will work in their class.

Overall Wrap Up

Note: this part will be done in a discussion format.

- What changes do you plan to make in your mathematics instruction?
- How will these changes benefit your students?
- How will you implement the Standards for Mathematical Practice into your selection of problems and lesson planning?
- What additional support do you need to help you make these changes?

Project/Homework

- For further reading: Kabiri and Smith, "Turning Traditional Textbook Problems into Open-Ended Problems," and Schwan and Smith, "Selecting and Creating Mathematical Tasks: From Research to Practice."
- After reading the articles, participants will create three open-ended problems to be shared with the group via email, Wiggio, etc. Participants may also be instructed to share lesson plans based on these open-ended problems with the group.