Writing

Innovative teachers can make writing an invaluable part of math instruction.

Marilyn Burns

One reason I chose mathematics for my undergraduate major was that it didn't require papers. Math homework called for solving problems or proving theorems, and that was just fine with me. Math and writing, like oil and water, seemed to have little in common. And for my first 20 years as a middle school and elementary school teacher, writing played no role in my math teaching.

Today, my view has changed completely. I can no longer imagine teaching math without making writing an integral aspect of students' learning.

This transition occurred over a period of years as I gradually overcame my own writing phobia. After writing children's books, professional books, and numerous articles, I've faced my writing dragon. I've even come to appreciate writing as a tool for clarifying my thinking. As Zinsser says, "Writing is a way to work yourself into a subject and make it your own" (1988, p. 16).

But the results I experienced with students were what clinched my commitment to making writing a regular part of math instruction. Not only did I see how writing helped students think more deeply and clearly about mathematics, but I also discovered that students' writing was an invaluable tool to help me assess their learning.

Writing in math class supports learning because it requires students to organize, clarify, and reflect on their ideas—all useful processes for making sense of mathematics. In addition, when students write, their papers provide a window into their understandings, their misconceptions, and their feelings about the content they're learning.

Of course, the writing that students do in math class generally differs from the writing they do in other subjects, such as language arts or social studies. Writing in math class isn't meant to produce a product suitable for publication, but rather to provide a way for students to reflect on their own learning and to explore, extend, and cement their ideas about the mathematics they study. Aside from the students themselves, I consider myself the primary audience for my students' writing, and I pay attention to what that writing shows about their math understanding—*what* they write, not *how* they write it.

A Variety of Writing Assignments

The writing assignments I give fall into four categories: keeping journals or logs, solving math problems, explaining

For Review Only

in Math

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mathematical ideas, and writing about learning processes. Each of these categories focuses the students on their math learning in a different way, and each provides useful information for assessing their progress.

Keeping journals or logs. When students create ongoing records about what they're doing and learning in math class, they have a chronological record of their learning experiences. I often post suggestions such as these to help students focus on their journal writing, especially at the beginning of the year:

- Write about what you did in class.
- What did you learn?
- What are you unsure about, confused by, or wondering about?
- Describe what was easy and what was difficult for you.

At times I give guidelines that relate to a specific lesson. For example, "Explain why Raul's answer made sense," or "Write about why Kaisha and Robert disagreed."

I organize students' journal writing in various ways. Sometimes I staple sheets of paper into booklets with construction-paper covers, giving students a new booklet—and thus a fresh start—several times a year. (Remember the pleasant school ritual of having new notebooks for courses at the start of each semester?) Sometimes I ask students to write on loose paper, and I file their work in folders. One year I gave each student a spiral-bound notebook of ¼-inch graph paper, an expensive investment but one that helped them graph with ease. Whatever the approach, I always keep student journals or logs in the classroom except when I take them home to read. I instituted this policy after too many students came to class without their logs.

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Solving math problems. Mathematics instruction should engage students in applying a variety of strategies for solving problems and also teach them to monitor and reflect on their problem-solving processes. Writing enhances both of these skills. Even when students work cooperatively on a problem-solving situation, it's beneficial to have each student write his or her own paper. Although the group work gives them access to one another's ideas, writing requires students to clarify their thinking.

Sometimes problems have one right answer, even though there are different ways to get to the answer or explain why the answer makes sense. For example, to provide experience with reasoning proportionally and applying fractional concepts, I gave 5th graders a problem about mixing red and yellow paint to get orange paint. I told them I had made two batches: Batch A had two parts red paint and two parts yellow paint, and Batch B had three parts red paint and two parts yellow paint. "How would the colors of the two batches compare?" I asked them. The students easily understood that B would be a darker orange.

Then I asked them to compare two additional batches of paint—a new Batch A, with two parts red and one part yellow, and Batch B, with the same three parts red and two parts yellow—and write about their solutions. The class was divided in their opinions: Some thought that the two batches would be the same color, and some argued that Batch B would be darker. Becky, for example, wrote about why the colors would be the same:

> There is always one more red than yellow, so B should have more parts, but A and B should be the same color.

David, however, reasoned correctly that

> A would be darker than B because there is twice as much red as yellow. In B there should be four parts red to make A and B the same color.

The ensuing lively discussion was useful not only for the students but also for me because it gave me a chance to assess their understanding.

Other problems have more than one possible correct solution. For example, I told 3rd graders that I had bought an item and received 35 cents in change. "What might I have bought and how much did it cost?" I asked them.

Student responses to this problem

For Reader Only

resulted in a situation that I hadn't counted on but that added an interesting dimension to the lesson. Sam, the first student I called on, said, "You could have bought some candy that cost 70 cents and paid with a dollar bill and a nickel." I told Sam that his solution was mathematically correct but that it didn't make sense as a real-life solution. "If I were buying something for 70 cents," I asked him, "why wouldn't I merely pay with the dollar bill and get 30 cents in change?" Sam and the other students were surprised by my response, but they saw the sense in my analysis.

Next, Kellie suggested buying candy that cost 35 cents. "You'd pay with 70 cents," she said. "What coins would I use to pay for the candy?" I asked her. After a moment, she said, "Two quarters and two dimes." I responded, "But I could use just one quarter and one dime to pay, and then I wouldn't need change." Kellie groaned.

Next I asked the students to talk at their tables about the answers they had found. In a few minutes, one table was sure they had an answer that was mathematically correct and sensible, too: Nathan reported that I could have bought a game for $9.65 and paid with a $10 bill. At another table, Maria said that I could have bought a sandwich for $4.65 and paid with a $5 bill.

Now the time was ripe for a writing assignment. I asked the students to record an answer and write about why it made sense in two ways—mathematically and also as a real-life solution. The students' writing helped cement for them the idea that they needed to consider answers to problems from different perspectives.

Explaining mathematical ideas. From time to time, I ask students to write an essay about a math concept. Their responses often provide useful information for assessing what they understand. I structure these assignments in a straightforward manner. For example, during a unit on probability, I asked 3rd graders to write about *equally likely,* explaining what it means and giving an example. At the end of a unit on division, I asked 4th graders to write about how multiplication and division were alike and different.

Photo courtesy of Marilyn Burns Education Associates

In the middle of a unit on fractions, I asked 5th graders to write on the topic "What I Know About Fractions So Far." (This title conveyed to them that I didn't expect them to know everything yet.) Nelson's response tipped me off to his misconception about the smallest possible fraction. He wrote,

> I know about halves and fourths. I know about eighths, too. And I know that sixteenths are the smallest fractions there are.

I had recently introduced Fraction Kits to the students in which they cut strips of construction paper into halves, thirds, fourths, sixths, eighths, and sixteenths and then used the strips for a variety of games and activities. Nelson had erroneously generalized from this limited set of fractions. This was important information for me to have. The next day, I had the students cut another strip into sixteenths and then cut each of the sixteenths in half. We talked about the fractional part of the whole these pieces represented and labeled them each $\frac{1}{32}$. We talked about continuing the pattern of cutting pieces into smaller pieces, resulting in fractions with larger and larger denominators. This seemed to help Nelson abandon his incorrect notion.

Writing about learning processes. Some writing assignments don't focus on a math problem or concept but instead on some general aspect of the students' learning during math class. For example, students might discuss their favorite and least favorite activities in a unit. Or they might write about the qualities of a good problem-solving partner in math class. Sometimes they write directions for an activity or a game so they can teach it to someone at home. I've also had students write letters describing the activities we do in class, to be used as an introduction for visitors before they observe one of our math lessons.

With this variety of assignments, I make writing a regular feature of math class. Although different students enjoy different writing assignments, I find that incorporating a mix of writing activities gives me broader insights into my students' math experiences.

Strategies for Incorporating Writing

My skill and comfort with making writing integral to math learning evolved through years of experience, which included many mistakes and discoveries. Here are some classroom strategies that I recommend to teachers who want to incorporate writing into their math instruction:

■ *Establish the purpose for writing in math class.* Make sure students

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understand the two basic reasons that writing is an important part of math: to support their learning and to help you assess their progress.

■ *Establish yourself as the audience.* Let students know that their writing will help you teach them better by providing valuable insights into their understandings, misconceptions, and confusions.

■ *Ask students to include details and to explain their thinking as thoroughly as possible.* Encourage them to use words, numbers, and, if they like, pictures to provide as much information as possible.

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■ *Have students discuss their ideas before writing.* Most students find talking easier than writing, and opportunities to talk about their thinking can help students formulate ideas that they will then try to explain in writing. Class discussions are useful, but having students talk in small groups enables more students to express their ideas. After a discussion, remind students that they can write about any idea they heard, as long as it makes sense to them and they can explain it.

■ *Post useful mathematics vocabulary.* Maintain a class chart showing pertinent mathematics vocabulary that comes up in class discussions. Before students begin work on an assignment, ask if there are other terms they might use that they'd like to see on the chart. Keep this list posted and add to it during the year.

■ *Write a prompt on the board to get students started on a writing assignment.* Sometimes a general prompt is appropriate: *I think that the answer is* _____. *I think that because* _____. Sometimes you'll want to use a prompt that relates more specifically to the assignment. For example, after asking 3rd graders how many chopsticks are needed for the 28 people in the room, I wrote on the board: *We need* ____ *chopsticks. I figured it out by* _____.

■ *Give individual assistance as needed.* No matter how thoroughly you prepare students for a writing assignment, some will need additional assistance to get started. Talk with these students first to ascertain that they understand the assignment. Then encourage them to verbalize their thinking by asking them such questions as, "What do you think? What's one idea that you have? What do you remember about what others said?" After students offer an idea, ask them to repeat it aloud. Then suggest that they repeat it silently to themselves. Finally, tell them to write those exact words on their paper. This process often helps jump-start students' writing. If it doesn't, try refocusing the assignment and asking students to write about what they find confusing.

■ *Have students share their writing in pairs or small groups.* Before handing in an assignment, students may benefit from reading their papers to another student or to a small group for feedback. This exercise not only serves the writer but also enables students to hear other points of view.

■ *Use students' writing in subsequent instruction.* Student papers can provide useful springboards for extending lessons, and using them in this way reinforces the idea that you value students' writing.

■ *Use student papers to create class inventories.* Reading a set of papers from an assignment can provide a useful overview of how the class responded to particular lessons. Was the experience accessible to all? Did it interest or challenge the more capable students? What additional instruction do students need?

■ *Keep each student's work in an individual folder.* After reviewing students' papers, file them in folders to create a chronological collection of work for each student. Reading through individual folders of work done over time provides a useful record of the student's progress. The folders can also be a great tool for parent conferences.

Taking on the Challenge

The National Council of Teachers of Mathematics has acknowledged writing as an important component of math instruction. The Council's *Principles and Standards for School Mathematics* (2000) states that "written communication should be nurtured" (p. 62) and that math instruction should enable all students to

■ Organize and consolidate their mathematical thinking through communication.

■ Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.

■ Analyze and evaluate the mathematical thinking and strategies of others.

■ Use the language of mathematics to express mathematical ideas precisely. (p. 60)

It's up to mathematics teachers to take on the challenge of implementing these goals in classroom instruction. You can begin by understanding how writing benefits your students' understanding of mathematics and by working to incorporate a broad variety of writing activities into your classroom. As you hone your repertoire of instructional strategies, you'll find that students come to accept writing as a reasonable extension of what they're doing in class and, finally, as a natural and integral part of their math learning. EL

References

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